STRUCTURAL BEHAVIOR OF REINFORCED CONCRETE DEEP BEAMS WITH AND WITHOUT OPENINGS

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ABSTRACT

The behavior of R.C deep beams is predicted using a proposed layered isoparametric finite element model. Most of the properties of concrete in compression and tension domains were considered in the model. The model was used to study the behavior of deep beams with and without openings taking into consideration progressive cracking of concrete, effect of shear-span to depth ratio, opening's size, location and skew (anisotropic) reinforcement around openings. The model was verified by comparing predicted behavior with experimental and theoretical results in the literature. It was found that the increase in shear-span to depth ratio lead to a general enhancement of the beam ductility, however, there was a reduction of its stiffness. In addition, the opening location and size had a great effect on the behavior of the beams. The provision of additional reinforcement around openings, especially anisotropic reinforcement perpendicular to major crack direction, had a significant effect on the behavior of reinforced concrete deep beams, namely, increasing their ultimate strength. Such additional reinforcement also affects the cracking propagation behavior throughout loading process.

السلوك الإنشائي للكمرات الخرسانية المسلحة العميقة المصمتة وذات الفتحات

ملخص البحث

في هذا البحث تمت دراسة سلوك الكمرات العميقة الخرسانية المسلحة باستخدام نموذج نظري للعناصر المحددة الأيزوبارامترية متعددة الطبقات. وفي هذا النموذج يتم تمثيل خصائص الخرسانة في الشد والضغط ويتم حساب اتجاهات الشروخ وكذلك يتم تمثيل حديد التسليح باتجاهاته المختلفة. تم تطبيق نظرية العناصر المحددة لدراسة تأثير نسبة بحر القص إلى العمق، أبعاد الفتحات ومواقعها وكذلك تأثير وجود حديد تسليح إضافي حول الفتحات. تم التأكد من دقة البرنامج المستخدم بمقارنة نتائجه بنتائج معملية ونظرية من دراسات سابقة. بدراسة الكمرات ذات الفتحات في الأعصاب تبين أن زيادة نسبة بحر القص إلى العمق تؤدي إلى زيادة ممطولية تلك الكمرات مع خفض قوة تحملها ووجد أيضا أن أبعاد الفتحات وأماكنها له تأثير كبير على تلك الخصائص. كذلك تبين أن حديد التسليح الإضافي وخاصة المائل (عمودي على اتجاه الشروخ الرئيسي) حول الفتحات له تأثير ملموس على سلوك تلك الكمرات حيث أنه يزيد من قدرتها على التحمل وكذلك يؤثر على نمو الشروخ خلال زيادة الحمل.

INTRODUCTION

Deep beams are structural elements that are usually found in tall buildings, tanks and diaphragms to transfer loads from one part of the structure to another. The ACI-318-95 code [1] defined deep beams as beams with span/depth ratio less than or equal to 5 for simply supported beams. Strength of deep beams and their geometrical properties are usually governed by shear, provided that the nominal amount of main flexural reinforcement is present. The common mode of failure of deep beams is characterized by the gradual propagation of diagonal cracks towards the support and loading points along the natural load path [2]. Openings are usually provided in such beams to give an access for utility ducts without further increase in ceiling headroom. As the utilization of such beams with or without opening increases, it becomes imperative that they are accurately analyzed and their design criteria is widely tested and established.

A number of analytical investigations have been conducted to study the behavior and collapse loads of deep beams with and without web opening. Kong and Sharp [3] proposed a semi-empirical formula while Ray and Reddy [4] suggested a comprehensive expression based on the Mohr-Coulomb failure criteria for predicting the ultimate load-carrying capacity of deep beams. The strut and tie models were applied to reinforced concrete deep beams by several investigators including Foster and Gilbert [5], Tan et. al. [6], Ali [7] and Taylor et. al. [8]. Nonlinear finite element analysis was widely carried out [9, 10, 11 and 12]. Zaatar and El-Attar [13] used finite element analysis for deep beams with web openings based on eight-node isoparametric element for concrete and link (truss) element for steel reinforcement. A different approach, based on nonlinear analysis of layered isoparametric element, was carried out by Di and Cheung [14] and Lewinski and Wojewodzki [15].

Recently, a layered isoparametric nonlinear finite element program was developed by the author at Civil Engineering Department of Zagazig University for the analysis of reinforced concrete plates and shells [16]. This technique considers concrete in tension, concrete in compression and steel reinforcement as different layers of different properties. The program was titled "NLFEALSE", Non Linear Finite Element Analysis of Layered Structural Elements. In this investigation, the developed program was used to study the structural behavior of reinforced concrete deep beams with and without openings.

The objective of this paper was, firstly, to calibrate program "NLFEALSE" in order to allow it to analyze deep beams. Secondly, to study the behavior of deep beams with and without openings. Variables such as opening location, size and shear span to depth ratio were included. The load-deformation relationships and crack patterns were used to predict the behavior of studied deep beams. Special attention was given to the effect of reinforcement anisotropy around openings on the behavior of such beams.

ANALYTICAL MODELLING

Finite Element Program "NLFEALSE"

Program NLFEALSE is based on a quadrilateral isoparametric 4-nodes element with five degrees of freedom per node. The material modeling simulates the reinforced concrete section as consisting of a number of layers, namely concrete layers throughout the thickness, while the steel reinforcement is smeared into equivalent steel layers. Each layer is assumed to be in a state of plane stress. The concrete layer model considers the elastoplastic behavior in compression, elastic brittle fracture behavior in tension, crack tension stiffening, compression softening, and rotating crack concept [14 and 15]. The steel reinforcement layer is modeled by an idealized bilinear curve identical in both tension and compression [12]. Detailed information regarding program NLFEALSE, including material models, failure criteria and rate of convergence can be found in reference [16].

Modified Kong and Sharp's Formula

The semi-empirical formula proposed by Kong and Sharp [3] for reinforced concrete deep beams was modified by Swaddiwudhipong and Shanmugam [2] to simulate openings, extra shear capacity, crack arrest and fracture toughness for fiber reinforced concrete deep beams more accurately. In addition, the formula takes into consideration the orientation of steel reinforcement (anisotropy) which was given a special attention in this investigation. The modified form for ultimate capacity, P_u, is written as follows:

$$\frac{P_u}{2} = C_1\, f_1\, f_2 \left(1 - 0.35\, \frac{X}{D}\right) f_t^{\,n}\, b^{`} D + \Sigma\, \lambda\, C_2\, A \left(\frac{y}{D}\right) \sin^2 \alpha$$

where:

C_1 = 1.4 for normal weight concrete
C_2 = 130 N/mm^2 for plain round bars and 300 N/mm^2 for deformed bars.
f_t = cylinder splitting tensile strength, N/mm^2
f_1 = $(1 - a_1)(1 - 1.667\, a_2)$, the reduction factor for the size of opening
f_2 = $\left[h + 2(k_2)^r \times \left\{ \frac{(k_1 - k_2)^2}{(a_1 x)^2 + (a_2 D)^2} \right\}^{1/2} \right] \leq 1$, the reduction factor for the extent of interruption of the opening on the natural load path (see Figure 1)
n = 1.1, the power factor reflecting the presence of steel fibers.
λ = 1.5 for web bars and 1.0 for main bars
h = $0.6 - 2\,k \geq 0.2$
r = 1.0 when the center of the opening is in the unloaded quadrant and 2.0 when it is in the loaded quadrant (see Figure 1)
$b^{`}$ = width of the beam (mm)
X / D = clear shear-span to depth ratio
A = area of an individual web bar (mm^2)
y and α = the depth and the angle (orientation) at which a typical bar intersects a potential critical diagonal crack (anisotropy of reinforcement).
a, b, a_1, a_2, k, k_1 and k_2 = coefficients defining the size and position of the openings as illustrated in Figure 1.

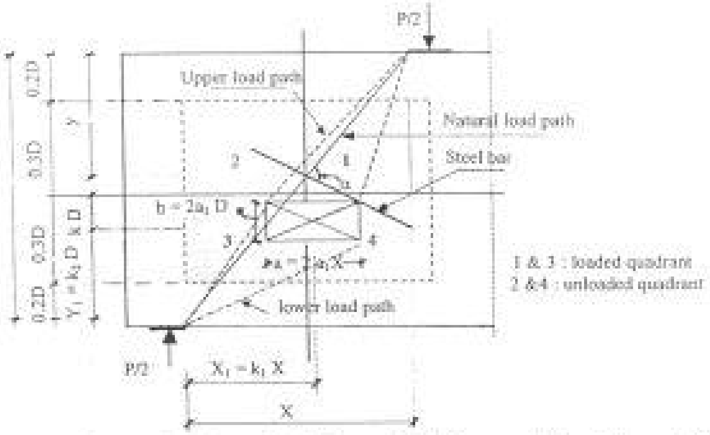


Figure 1 Structural configuration for the modified Kong and Sharp's formula [2]

SIMPLY SUPPORTED SOLID DEEP BEAM

A reinforced concrete deep beam, tested experimentally by Leonhardt and Walther [17], and modeled theoretically by Vecchio [9], was remodeled by program NLFEALSE in order to double check its accuracy. The beam was square in shape with span and height of 1600 mm, thickness of 100 mm, was simply supported at the bottom edge, and was subjected to a uniformly distributed load along its top edge (see Figure 2). Vertical reinforcement was uniform throughout (ρ_y =0.00175). The horizontal reinforcement was heavier in the lower regions (ρ_x=0.01787) and lighter above (ρ_x=0.00175) (see Figure 2). The material properties are reported in Table (1). Using symmetry, half the beam was modeled by Vechio [9] using 128 rectangular elements while NLFEALSE used only 32 quadrilateral isoparametric layered elements for modeling the same half beam as shown in Figure 3(a). The structure is considered as a membrane element and the reinforcement was smeared into a number of layers. Two layered models were considered in the analysis of the studied deep beam depending on the position of steel reinforcement (sections type I and type II) as shown in Figure 3(b).

Table 1 Material properties of concrete and steel reinforcement used in the studied deep beam

Concrete	Steel reinforcement
E_c = 20000 Mpa	E_s = 200000 Mpa
ν = 0.20	
f'_c = 29.6 Mpa	f_y = 415 Mpa
f'_t = 1.76 Mpa	5 mm bars, As = 20 mm^2
ϵ_o = 0.002	8 mm bars, As = 54 mm^2

Prediction of Mid-span Deflection

Figure 4 shows the observed response of the studied beam in terms of the mid-span deflection against the total applied load. The predicted response using NLFEALSE also shown in Figure 4. It can be seen from the figure that there is a good agreement between the results obtained by NLFEALSE and those obtained by Vecchio [9] despite the fact that NLFEALSE used only ¼ the number of elements used by Vecchio [9]. This may be attributed to the high accuracy of the proposed layered approach which

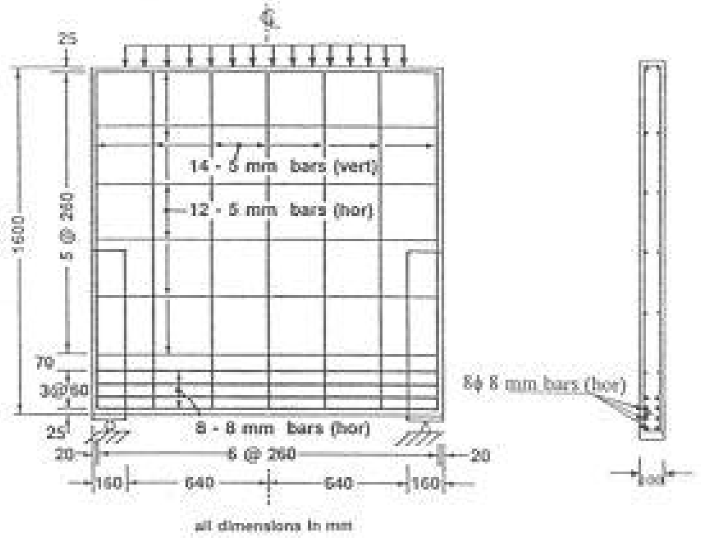
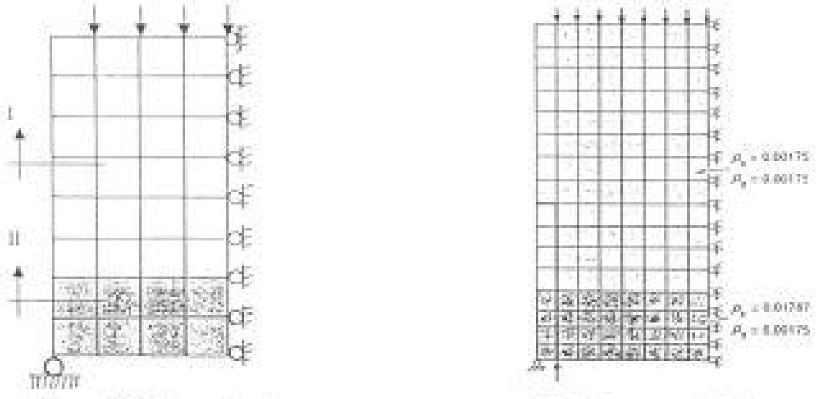


Figure 2 Dimensions, Reinforcement, Loading, and Boundary conditions of Leonhardt and Walther studied deep beam [17]



Proposed finite element mesh

Finite element mesh [9]

(a) Loading and boundary conditions for finite element analysis

Section type I

Section type II

(b) Proposed layered model

Figure 3 Finite element idealization and layered models of studied solid deep beam takes most of concrete properties in compression and tension domains into consideration.

Prediction of Crack Patterns

The crack pattern predicted by NLFEALSE is given in Figure 5 along with the experimental crack net presented by Leonhardt and Walther [17] at load level 800 KN. It was observed experimentally that, at approximately 37% of the ultimate load, yielding occurred in the support area. At 55% of ultimate load, first tension cracks were formed at the bottom of the beam. With the increase of loads, further yielding and cracking occurred and the ultimate strength was reached at a load of 1200 kN when material instability was obtained due to yielding and cracking in the support areas [17]. It can be seen from the figure that the analytical prediction was in good agreement with the experimental behavior observed by Leonhardt and Walther [17] at load of 800 kN.

DEEP BEAMS WITH WEB OPENINGS

Figure 6 shows a simply supported fiber reinforced concrete deep beam with overall length of 1550 mm (clear span of 1300 mm) and cross-section dimensions of 80×675 mm with two openings symmetrically placed. This was a typical beam in the series tested experimentally by Swaddiwudhipong and Shanmugam [2]. The compressive strength of the standard cube, f_{cu}, ranged from 400 to 450 kg/cm^2. Two 16 mm diameter deformed bars of 410 MPa (4182 kg/cm^2) yield stress were used as the main tension reinforcement. The bars were anchored by welding to 10 mm thick steel plates at both ends. The beams contained web reinforcement consisting of two layers of welded wire fabric of 3.3 mm diameter and 50 mm on centers having a yield stress of 300 MPa (3060 kg/cm^2) [2].

A total of nine beams were analyzed numerically using NLFEALSE and their ultimate strengths were predicted theoretically by the modified form of Kong and Sharp's formula [2 and 18] shown above. The studied variables were shear-span to depth ratio, opening location and size. Detailed description of the variables is shown in Figure 7 and Table 2. It is worth mentioning that 5 of the analyzed beams, namely Beams No. 1, 3, 5, 7 and 9 were tested experimentally by Swaddiwudhipong and Shanmugam [2].

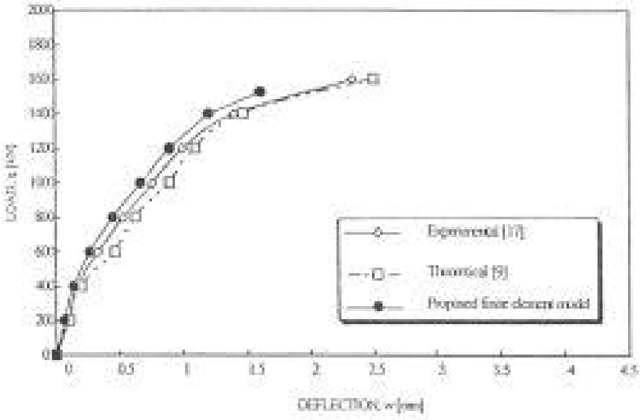


Figure 4 Load-deflection relationships of studied solid deep beam

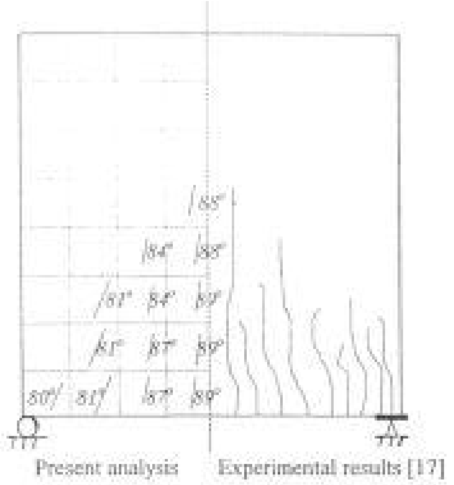


Figure 5 Predicted and observed crack patterns of studied solid deep beam (at load level 800 kN)

Using NLFEALSE and taking symmetry into consideration, only half of each beam was modeled. The proposed finite element mesh, boundary conditions, loading, reinforcement idealization, and layered models for different cross sections are shown in Figure 8.

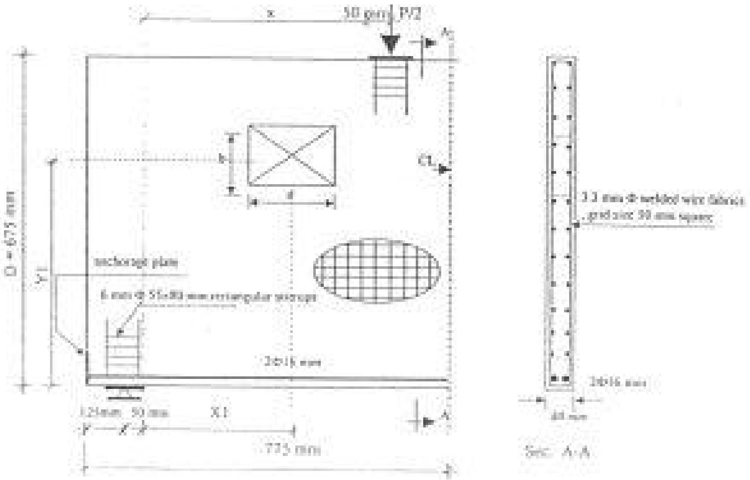


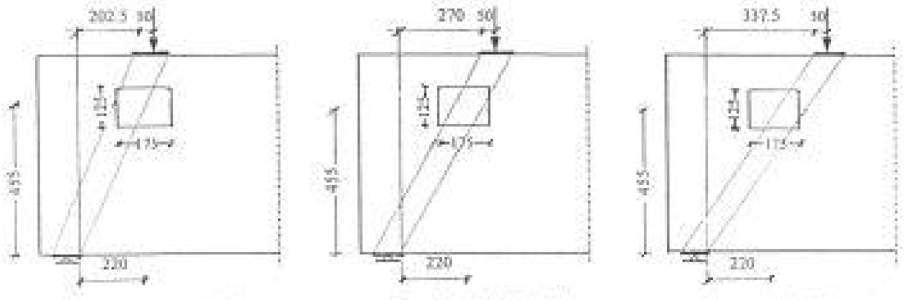
Figure 6 Dimensions and reinforcement bars of the studied deep beams with web openings [2].

Table 2 Variables considered in the analysis*

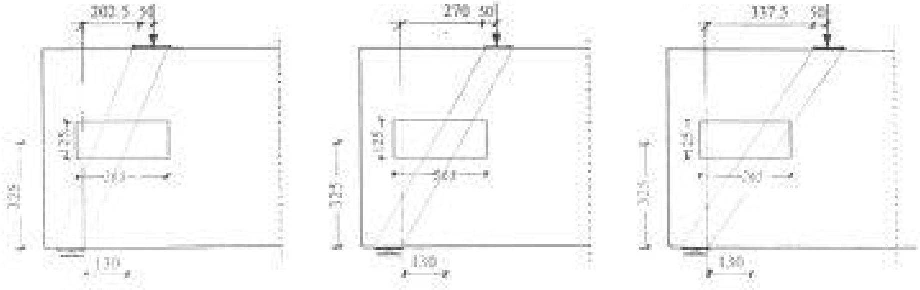
Beam Number	Shear Span to depth ratio			Opening Location		Opening Size	
	X	D	X/D	X_1	Y_1	a	b
1**	202.5	675	0.3	220	455	175	125
2	270	675	0.4	220	455	175	125
3**	337.5	675	0.5	220	455	175	125
4	202.5	675	0.3	130	325	265	125
5**	270	675	0.4	130	325	265	125
6	337.5	675	0.5	130	325	265	125
7**	202.5	675	0.3	40	195	175	125
8	270	675	0.4	40	195	175	125
9**	337.5	675	0.5	40	195	175	125

* *All dimensions are in mm*

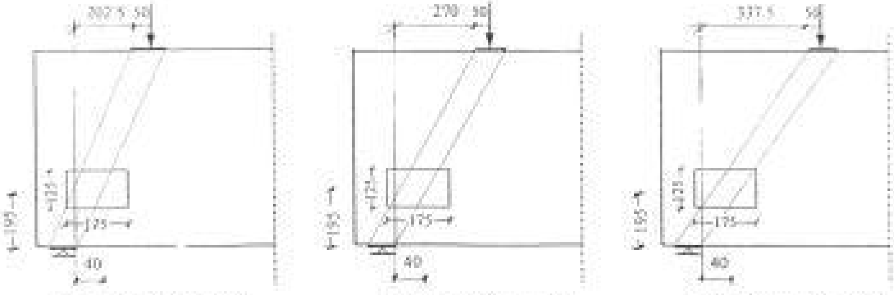
** *Experimentally tested by Swaddiwudhipong and Shanmugam [2]*



Beam No. 1 (X/D = 0.3) Beam No. 2 (X/D = 0.4) Beam No. 3 (X/D = 0.5)



Beam No. 4 (X/D = 0.3) Beam No. 5 (X/D = 0.4) Beam No. 6 (X/D = 0.5)



Beam No. 7 (X/D = 0.3) Beam No. 8 (X/D = 0.4) Beam No. 9 (X/D = 0.5)

Figure 7 Details of opening size, location and shear-span to depth ratio (X/D).

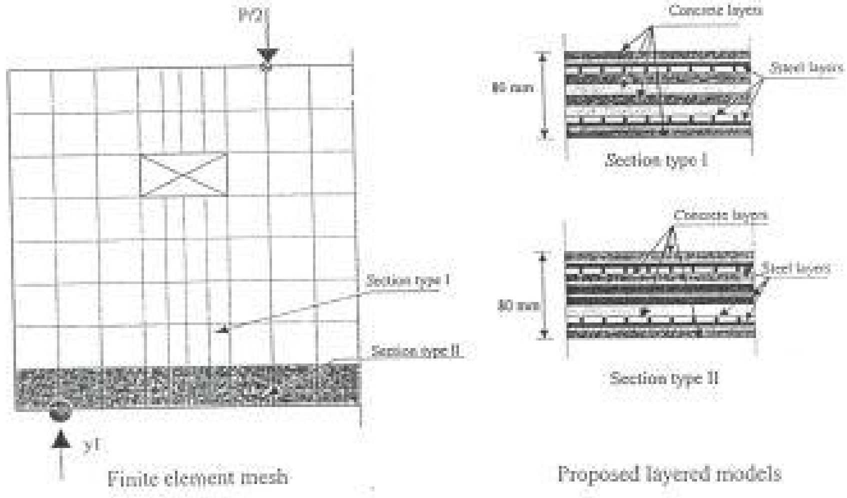
Prediction of Mid-span Deflection

The experimental results of mid-span deflection for Beams No. 1, 3, 5, 7 and 9 of different opening locations, sizes and shear-span to depth ratio, were predicted by program NLFEALSE as shown in Figure 9. It can be seen from the figure that, generally, there is a good agreement between the experimental results [2] and those predicted by NLFEALSE except for Beam No. 1, where the load punched through the opening during the experimental testing [2]. It was observed also that the stiffness of the studied beams is affected by the shear-span to depth ratio (X/D) as well as the opening size and location. For example, Beam No. 7 of X/D = 0.3 is stiffer than Beam No. 9 of X/D = 0.5 resulting in lower deflection values. In addition, Beam No. 9 had higher deflection values than Beam No. 3 despite the fact that both of them have the same X/D. This is probably because of the location of the opening in Beam No. 9, which lies near to the tension zone (see Figure 7).

Behavior of Deep Beams with Different Shear-Span To Depth Ratio, Opening Sizes and Locations

Figure 10 shows the load-deflection relationships predicted by NLFEALSE for the studied beams shown in Figure 7 and described in Table 2. Generally, it is shown that increasing X/D leads to enhancement of the ductility and reduction of stiffness (capacity) of studied beams, which have the same opening size and location. This is similar to the findings of Tan and Lu [19]. However, the effect of X/D is dependent on opening location and size, or in other words, on the degree of interruption of the openings to the natural load path as shown in Figure 7.

For example, Beam No. 1 of X/D = 0.3 had a mid-span deflection of 2.7 mm at load of 300 kN while Beam No. 4 of the same X/D achieved the same deflection value at load of 260 kN only. This may be attributed to the fact that the opening size of Beam No. 4 is larger than that of Beam No. 1, which results in a reduction of the stiffness of Beam No. 4 compared to Beam No. 1. Moreover, the deflection of Beam No. 3 (X/D = 0.5) was 2.3 mm at 200 kN while the deflection of Beam No. 9 of the same X/D and the same opening location (see Figure 7) was 2.7 mm at the same load level. It can be argued that the stiffness of the beam was reduced when the opening was located near to the tension zone.



Finite element mesh

Proposed layered models

Figure 8 Proposed finite element mesh and layered models of the studied deep beams with web openings (Beam No. 3).

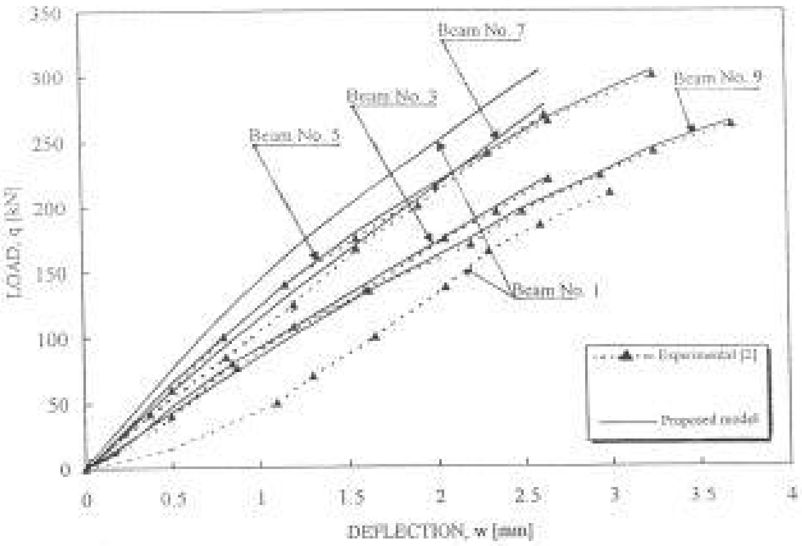


Figure 9 Prediction of load-deflection relationships for studied deep beams with web openings.

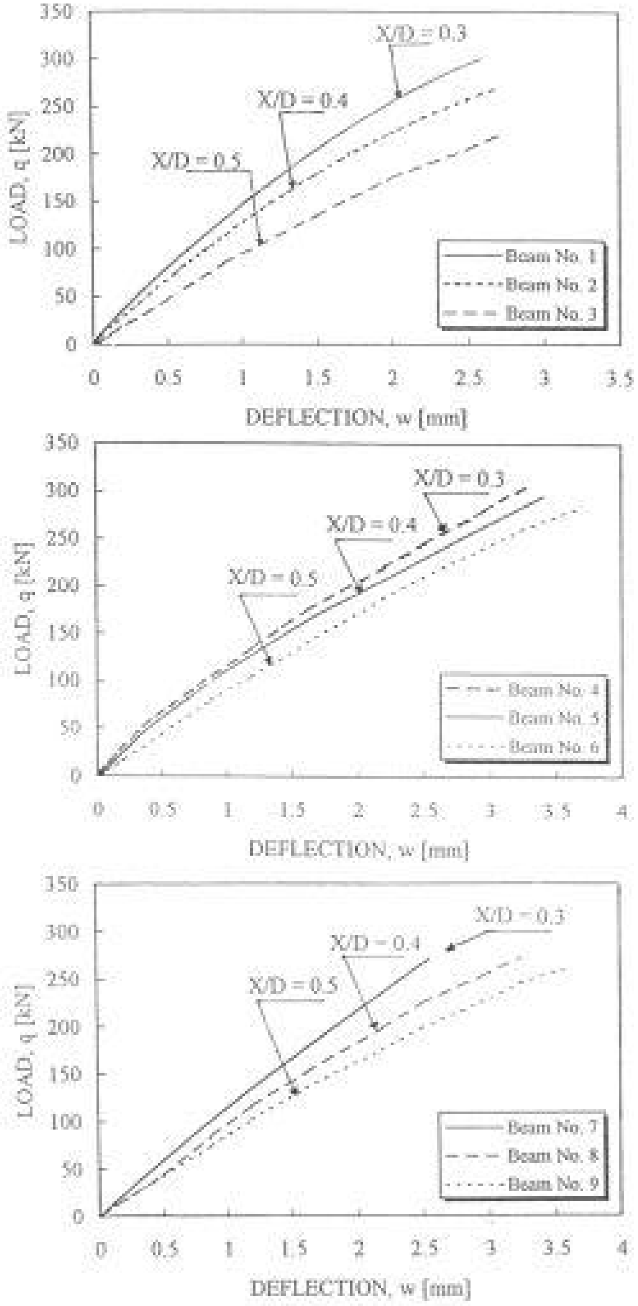


Figure 10 Effect of opening location, size and shear-span to depth ratio on the behavior of deep beams with web openings.

Prediction of Ultimate Shear Capacity

Experimental load-deflection readings were taken up to 205, 220, 300, 265 and 260 kN for Beams No. 1, 3, 5, 7 and 9 as shown in Figure 8, whereas the beams failed at 288, 350, 340, 310 and 280 kN, respectively [2]. It was observed experimentally that the mode of failure was diagonal shear [2]. Foster and Gilbert [5], Tan and Lu [19] and Ashour [20] found that the ACI Building Code [318-89] gave conservative results for the expected failure loads. Moreover, Foster and Gilbert [5] reported that CIRIA design model, which is originally based on Kong and Sharp's formula [3], is accurate for small shear-span to depth ratios. For deep beams with openings, Swaddiwudhipong and Shanmugam [2] found that the modified form for Kong and Sharp's formula is the most accurate and simple empirical formula.

Table 3 shows a comparison between the ultimate load capacity obtained experimentally for the studied deep beams, those predicted by the modified Kong and Sharp's formula [18] and those predicted by NLFEALSE. Within the available limited number of experimental tested beams, it can be seen that the experimental results are well predicted by NLFEALSE as well as by the modified Kong and Sharp's formula [18].

Table 3 Prediction of ultimate shear capacity

Beam Number	Ultimate Strength, Kn		
	Experimental results [2]	Modified Kong and Sharp's formula [18]	Proposed finite element model
1	288*	365	392
2	----	344	372
3	350	321	360
4	----	384	375
5	340	364	350
6	----	338	325
7	310	325	316
8	----	306	303
9	280	274	288

* The beam failed by punching shear

Crack Patterns at Different Load Stages

Figure 11 shows crack patterns obtained numerically by NLFEALSE for Beam No. 3. It was noticed that computed results predict closely the failure load of 360 kN (see Table 3). Based on experimental observation reported by Swaddiwudhipong and Shanmugam [2], a typical sequence of crack propagation till ultimate load for beams with different opening sizes and locations and variable shear span ratios is depicted in Figure 12. It can be seen from the figure that the major direction of cracks around the opening is in the same load path direction shown in Figures 1 and 7. Figure 12 shows that the first crack was usually a suddenly inclined shear crack originating from the outer bottom corner of the opening (crack 1). With further increase of load, crack 1 propagated downward towards the support while crack 2 originating at the top inner corner of the opening propagated upward towards the load bearing plate. Other flexural and flexural-shear cracks (cracks 3 and 4) were subsequently formed and progressed upwards. At higher loads, diagonal cracks 6 and 7 developed and propagated until the beam failed in diagonal shear mode [2]. Reliable predictions can be seen, with all major features of experimental observations being reproduced.

Effect of Anisotropy of Additional Reinforcement Around Openings

Based on the crack pattern shown in Figures 11 and 12, a theoretical attempt to improve the behavior of deep beams with openings, which were experimentally tested and described earlier in Figures 6 and 7, by introducing additional reinforcement around the openings, was carried out numerically by the program NLFEALSE. The additional reinforcement was in the form of parallel bars to the edges of the opening and anisotropic (skewed) around openings as shown in Figure 12. It was divided in the finite element modeling, with respect to reinforcement orientation angle, into four zones (see Figure 13).

(a) Orientation of Anisotropic Reinforcement

It was found after several attempts using NLFEALSE that the direction of diagonal bars perpendicular to the load path direction (major direction of cracks around openings) achieves the maximum improvement to the beam capacity. It can be seen from Figures 1 and 7 that the load path direction is dependent on the shear-span to depth ratio. This

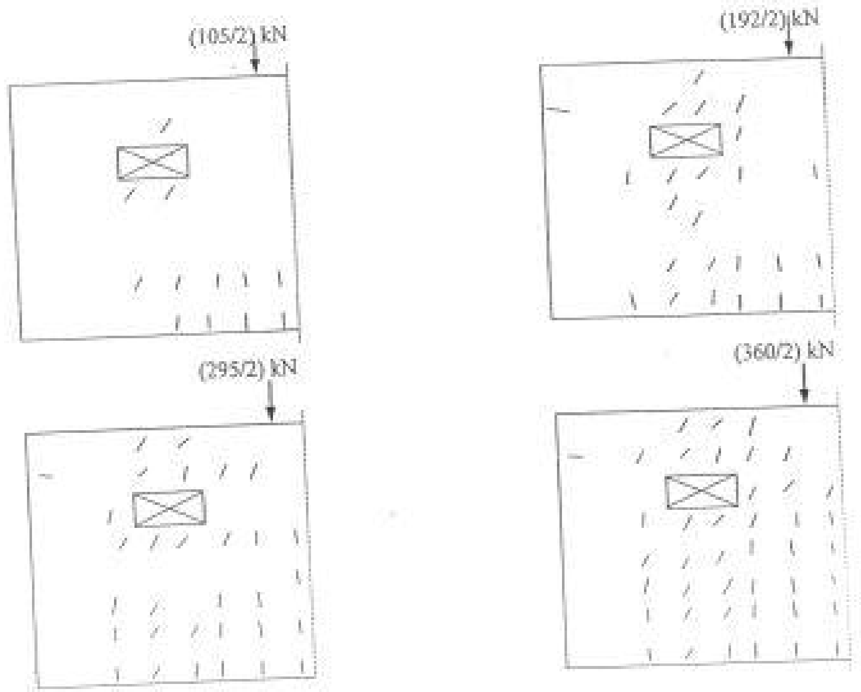


Figure 11 Crack pattern development during analysis for Beam No. 3.

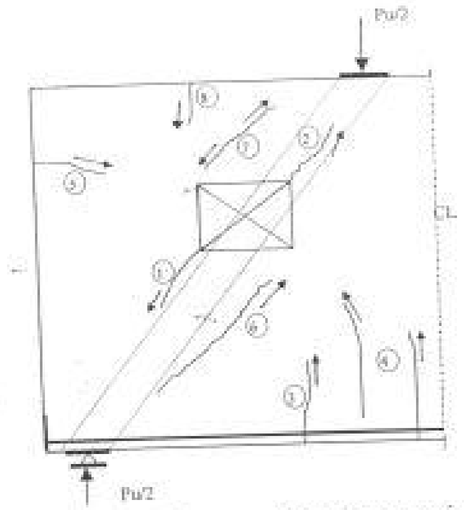


Figure 12 Typical sequence of cracks propagation [2].

was supported by the modified Kong and Sharp's formula reported earlier in this investigation. The formula showed that the capacity of beams reaches its maximum when the angle (α) between diagonal bar and load path becomes 90° (see Figure 1).

(b) Load-Deflection Relationships

Figure 14 shows load-deflection relationships for Beam No. 3 before and after providing additional reinforcement around the openings in order to check the sensitivity of the program NLFEALSE in simulating the anisotropically reinforced elements. It can be seen from the figure that there is a general improvement in the behavior of the deep beam at all load stages especially for high load levels. The additional reinforcement increased the capacity of the beam by 29%. It can be argued that the additional reinforcement, either parallel to the edges or anisotropically distributed around corners, increased the stiffness of the beam and this, in turn, resulted in delaying the propagation of diagonal cracks around openings, shown in Figure 12.

(c) Ultimate Strength of Beams

Figure 15 shows the ultimate strength of the studied nine beams before and after adding anisotropic reinforcement around openings. The figure shows that the additional steel improves the behavior of studied beams to different levels ranged from 23% to 35% based on shear-span to depth ratio, opening location and size. The computed improvement is inversely proportional to the ultimate strength of the studied beam (see Figure 16). For example, the ultimate strength of Beam No. 1, which had an original predicted strength of 392 kN and X/D = 0.3 was improved by 23% only while that of Beam No. 9, which had an original predicted strength of 288 kN and X/D = 0.5, was improved by 35% as shown in Figure 16. It can be argued that the original stiffness of Beam No. 9 is low since the opening lies near to the tension zone and in the loaded quadrant (see Figures 1 and 7). Therefore, adding anisotropic reinforcement improves the stiffness of such beam significantly.

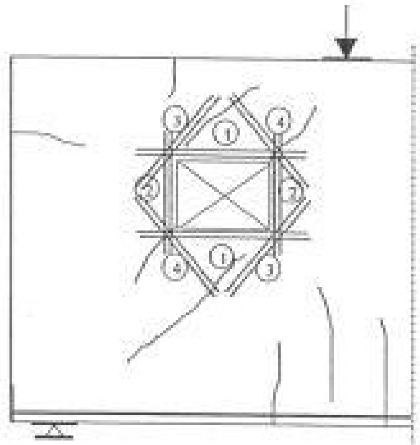


Figure 13 Arrangement of additional reinforcement around the openings

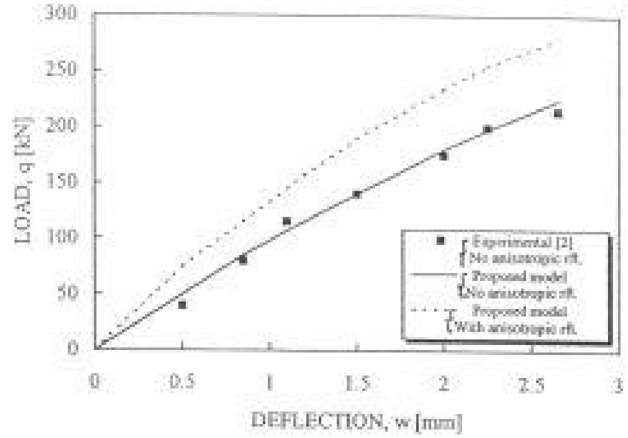


Figure 14 Load-deflection relationships for Beam No. 3 before and after providing additional reinforcement around openings.

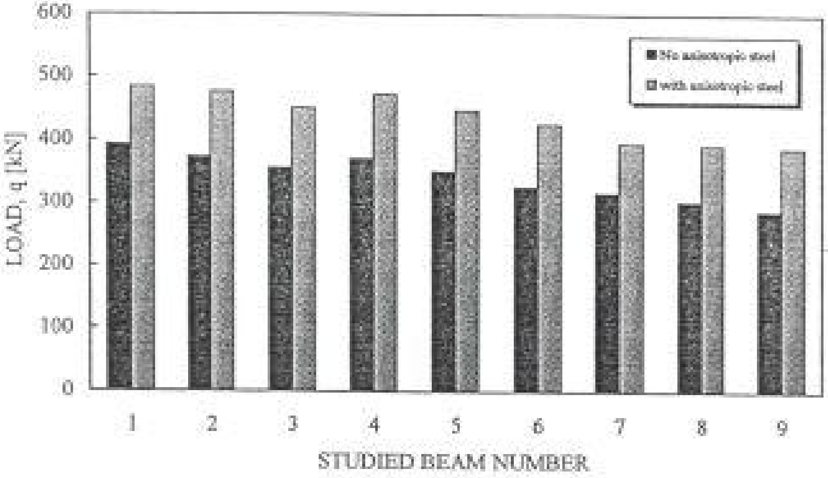


Figure 15 Effect of adding anisotropic reinforcement around openings on the ultimate capacity of studied beams

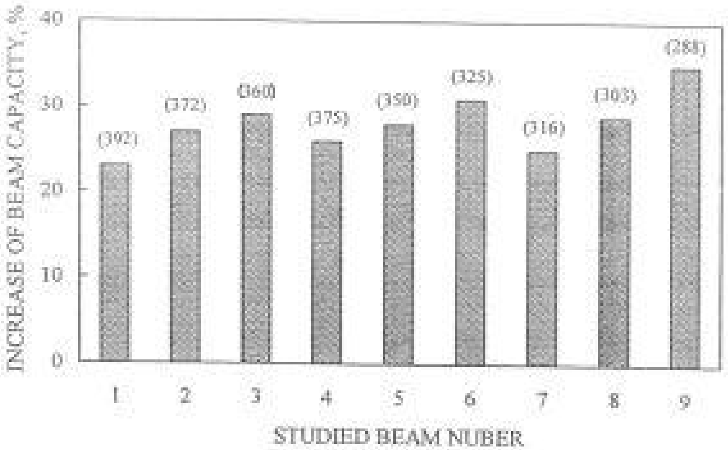


Figure 16 Relationship between studied beam number (original strength) and increase of capacity %

CONCLUSIONS

A nonlinear finite element program titled "NLFEALSE" was used to predict the response of reinforced concrete deep beams. The cracked concrete and steel reinforcement were effectively treated as anisotropic layers in the program. NLFEALSE proved to be capable of predicting the response of solid deep beams or those with openings. The obtained crack patterns and load-deflection relationships were consistent with the experimentally recorded results and also with theoretical ones.

Increasing the shear-span to depth ratio lead to enhancing the ductility and reducing the stiffness of studied deep beams with openings. However, the effect of shear-span to depth ratio is dependent on the opening location and size.

The presence of anisotropic (skew) additional reinforcement around openings increased the capacity of studied beams by 23-35%. The original strength is inversely proportional to the increase of capacity. Program NLFEALSE simulated the skew reinforcement easily as anisotropic smeared layer. Both of the program NLFEALSE and modified Kong and Sharp's formula proved that the best direction of anisotropic reinforcement is the perpendicular to the load path (major crack direction), which in turn, is dependent on shear-span to depth ratio.

The results obtained by the modified Kong and Sharp's formula provides accurate prediction of the behavior of reinforced concrete deep beams with openings. It can be used to initiate the analysis and design process prior to complete analysis using accurate finite element modeling such as NLFEALSE.

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